Basic Connection between Superconductivity and Superfluidity

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A basic and inherently simple connection is shown to exist between superconductivity and superfluidity. It is shown that the author's previously derived general equation, which agrees well with the superconducting transition temperatures for the heavy-electron superconductors, metallic superconductors, oxide superconductors, metallic hydrogen, and neutron stars, also works well for the superfluid transition temperature of 2.6 mK for liquid ^{3}He. Reasonable estimates are made from 10^{-3} to 10^9 K—a range of 12 orders of magnitude. The same paradigm applies to the superfluid transition temperature of liquid ^{4}He, but results in a slightly different equation. The superfluid transition temperature for dilute solutions of ^{3}He in superfluid ^{4}He is estimated to be ~ 1–$10\,\mu$K. This paradigm works well in detail for metallic, cuprate, and organic superconductors.

1. INTRODUCTION

The experimental discovery in 1972 of superfluidity in liquid ^{3}He (L^{3}He) at 2.6 mK was long preceded by predictions of the critical temperature T_c for this transition at $\lesssim 0.1$ K, which was then well below the lower limit of the known experimental data. These predictions were based on the BCS theory (Bardeen *et al.*, 1957), where the sensitive exponential dependence of T_c makes it hard to make accurate predictions of T_c. Theoretical papers (Pitaevskii, 1959; Brueckner *et al.*, 1960; Emery and Sessler, 1960) incorporated the pairing of ^{3}He Fermi atoms to make bosons by analogy with the Cooper pairing of Fermi electrons in metals. The theoretical and experimental ^{3}He work are thoroughly discussed with ample references in excellent review papers respectively by Leggett (1975) and Wheatley (1975). Betts' (1969) excellent tutorial review paper covers both superconductivity and only the superfluidity of ^{4}He, as superfluidity had not yet been discovered

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in ^{3}He. However, Betts does discuss dilute solutions of ^{3}He in ^{4}He as the best and then most recently realized examples of Fermi degeneracy.

When the experimental work showed that T_c must be well below the theoretical prediction of 0.1 K, theoreticians pushed $T_c \sim 10^{-9}$ K—then beyond the hope of experimental verification. Agreement of the theory with experiment was established after the experimental detection by Osheroff *et al.* (1972*a*) of remarkable features ≤ 3 mK of the pressurization curve (pressure vs. time) of L^{3}He in equilibrium with solid ^{3}He at ≈ 34.4 bar. Originally, they interpreted this in terms of effects in the solid ^{3}He. Their subsequent NMR experiments established that the effects were in the L^{3}He (Osheroff *et al.*, 1972*b*) with $T_c \leq 3$ mK for the superfluid transition.

2. ANALYSIS FOR HELIUM-3

As the temperature is decreased in a Bose–Einstein (BE)-like gas, particles should start a BE-type condensation (transition) into a ground state, i.e., the superfluid state when the thermal wavelength λ_T is comparable to the interparticle spacing d: $\lambda_T = h/(2\pi mkT_c)^{1/2} \sim d$. Here h is Planck's constant, m is the boson mass, k is the Boltzmann constant, and T_c is the critical (transition) temperature. I prefer the de Broglie wavelength λ rather than λ_T for reasons of clarity and comparison with my previous work (Rabinowitz, 1987, 1988, 1989*a,b*, and 1990). As shown in Figure 1, $\frac{1}{2}\lambda$ encompasses the centers of mass of two pairs of fermions when

$$\lambda \gtrsim 4d = 4n_s^{-1/3} \tag{1}$$

where n_s is the number density of particle pairs which can have an effective interaction for condensation because they have energies within kT_c of the
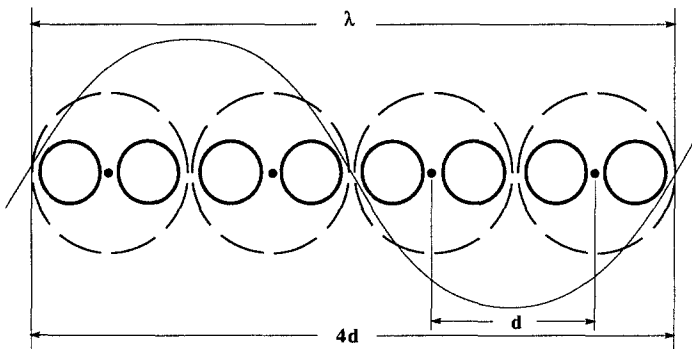

Fig. 1. ^{3}He fermion pairs.

Fermi surface,

$$n_s \sim \frac{kT_c}{E_F} n \qquad (2)$$

where n is the number density of fermions (^{3}He in the particular case we are considering) in the normal state, and E_F is the three-dimensional Fermi energy,

$$E_F = \frac{h^2}{2m_e} \left(\frac{3n}{8\pi}\right)^{2/3} \approx \frac{h^2}{8m_e} n^{2/3} \qquad (3)$$

where m_e is the effective mass of the fermion. In the case of ^{3}He, the effective mass m_e may be due to a screening cloud of additional atoms which act like a quasiparticle. This is not unlike the classical effective mass of a body moving through a liquid, which also moves some of the liquid along with it. The quasiparticles fill the Fermi sea up to the Fermi momentum.

For a particle pair of momentum p and mass $2m_e$,

$$\lambda = \frac{h}{p} = \frac{h}{[2(2m_e)(\frac{1}{2}fkT_c)]^{1/2}} \qquad (4)$$

where f is the number of degrees of freedom per particle pair. For three-dimensional ^{3}He, we will simply take $f=3$. See Rabinowitz (1989a) for a more general discussion of f.

Combining equations (1)–(4), we find the transition temperature

$$T_{cF} \leq \frac{h^2 n^{2/3}}{(8f)^3 m_e k} \qquad (5)$$

In general, equation (5) applies to paired fermion particles forming a boson gas, be they electrons or atoms. This is the expression derived as equation (5) in Rabinowitz (1989a). However, due to a misprint, the $(8f)^3$ was printed incorrectly as $8f^3$. Equation (5) gave a reasonable estimate for the transition temperatures of heavy-electron superconductors, metallic superconductors, oxide superconductors, metallic hydrogen, and neutron stars. It thus did a good job in covering a range of nine orders of magnitude from ~0.5 to 10^9 K. For three-dimensional L^{3}He, $f=3$. (As the dimensionality is lowered to quasi-2 or quasi-1 dimensions, the effective mass may change.) The effective mass and spin degeneracy should be used in conjunction with a more precise derivation. For now the free mass will suffice.

We shall now see that equation (5) also works well for the superfluid transition temperature of liquid ^{3}He. We need only two parameters, which are experimentally determined. Atkins (1959) gives the density of L^{3}He as

0.082 g/cm^3 at $T < 1$ K. This implies that the number density of L^{3}He atoms is $n \approx 1.6 \times 10^{22}$/cm^3. Taking $m_e \approx m_{3_{He}} \approx 5 \times 10^{-24}$ g, equation (5) gives

$$T_{cF} \approx 2.8 \text{ mK}$$

in excellent agreement with the experimental value of 2.6 mK at ≈ 30 bar pressure as given by Wheatley (1975). T_c decreases by a small factor with decreasing pressure, and there are two superfluid phases in L^{3}He.

In the ^{3}He-A phase, the nuclear spins of the two ^{3}He atoms are parallel to each other and tend to be perpendicular to the axis of orbital motion. In the ^{3}He-B phase the correlation is more abstruse. There are regions of pressure and temperature where each phase exists separately. Because all the pairs of a given kind are in the same state, the spin and orbital motion correlations of each kind exist throughout the superfluid as a whole. Thus, unlike superfluid ^{4}He, which has zero spin and is thus insensitive to perturbations which allow it to remain in the superfluid state, superfluid ^{3}He has an orientation which can be affected by external factors such as flow motion, interaction with surfaces, and applied magnetic and electric fields (Wheatley, 1975; Leggett, 1975).

3. COMPARISON BETWEEN LIQUID HELIUM-3 AND SUPERCONDUCTORS

The creation of superfluidity by the pairing of ^{3}He Fermi atoms to make bosons is analogous to the Cooper pairing of Fermi electrons in metals to create superconductivity. However, there is one important difference. In metallic superconductors, the paired electrons form a singlet state of zero spin (relative angular momentum), whereas the strong hard-core repulsion of ^{3}He atoms causes them to pair in a triplet state of spin 1. In the heavy fermion superconductors, it is possible that the electrons also pair in a triplet state of spin 1.

It is interesting to compare kT_c/E_F for L^{3}He and the different classes of superconductors, as this is a measure of the relative participation of potential carriers near T_c. Let us look at them in increasing order for T_c. For L^{3}He; $kT_c/E_F \sim 22$ μeV/44 meV $\sim 5 \times 10^{-4}$. For the heavy fermion superconductors; $kT_c/E_F \sim 0.1$ meV/1 eV $\sim 10^{-4}$. For the metallic superconductors; $kT_c/E_F \sim 1$ meV/10 eV $\sim 10^{-4}$. For the ceramic oxides; $kT_c/E_F \sim 10$ meV/2 eV $\sim 5 \times 10^{-3}$. For metallic H, it is expected that $kT_c/E_F \sim 20$ meV/4 eV $\sim 5 \times 10^{-3}$. For a neutron star, it is expected that $kT_c/E_F \sim 0.1$ MeV/200 MeV $\sim 5 \times 10^{-3}$. To first order it is remarkable how close all of these diverse quantum fluids are in their values of kT_c/E_F.

4. PREDICTIONS FOR LIQUID HELIUM-3 IN SUPERFLUID HELIUM-4

It is difficult to observe superfluidity in most substances because they go into the solid state before the extremely low temperatures are reached at which they might become superfluids. However, the combination of ^{3}He in superfluid ^{4}He does not have this problem, and allows for a prediction in a novel system for which superfluidity has not yet been observed.

According to my paradigm, if the temperature is lowered sufficiently, a dilute solution of ^{3}He atoms in superfluid ^{4}He should become a quantum fluid when the ^{3}He quantum wavelength is more than four times the ^{3}He interparticle spacing. This is analogous to the pairing of electrons in a metallic superconductor, where the coherence length is quite large, $\sim 10^4$ Å for a pure superconductor. The pairing of ^{3}He atoms in L^3He is more analogous to the pairing of electrons in a ceramic oxide superconductor where the coherence length is quite small, approximately a few angstroms. At close distances (high concentration), because of the hard-core repulsion, only the triplet state of parallel spins seems possible for ^{3}He. However, at low concentrations with large distances between the atoms, the singlet state of zero spin (antiparallel spins for the two atoms) would be energetically preferred. At intermediate concentrations, both singlet and triplet pairs may form. With the singlet state and ^{3}He-A and ^{3}He-B for the triplet state, seven combinations would be possible for this three-component superfluid.

As an analogue to the metal lattice, ^{4}He has the advantage not only of not becoming solid (which would impede the motion of ^{3}He) at these extremely low temperatures, but also of being a superfluid itself. The ^{4}He environment may act like a vacuum with negligible perturbation on the ^{3}He. It may even be the analogue of a solid superconductor in which paired electrons move like a charged superfluid. However, ^{3}He in ^{4}He interactions may be more important, as there may be many strong hard-core interactions in the scattering of the ^{3}He atoms in the ^{4}He environment. These collisions may reduce the coherence and hence T_c. In a way superfluid ^{4}He may to first approximation act like a kind of vacuum with mass with respect to ^{3}He atoms. This is not too unlike a contemplation by Finkelstein (1988, 1989) regarding the properties of space-time. The two-fluid (normal and super-states) interpenetrating model of superconductivity and of superfluidity works quite well. Rodríguez-Núñez and Tello-Llanos (1991) have recently developed a two-fluid model using the Dirac formalism.

Only dilute solutions of up to 10% ^{3}He in ^{4}He are possible. We shall here concern ourselves only with the volume displacement due to the foreign particles—^{4}He in this case. A sizeable correction must be made for a volume reduction effect due to the presence of ^{4}He. The corrected number density

η of ^{3}He in ^{4}He is

$$\eta = \frac{N_3}{V_3} = \frac{gN}{g'V} = \frac{g}{g'} n \approx n$$

where n is the overall number density, N_3 is the number of ^{3}He atoms, and V_3 is the volume available to them. For simplicity, I assume that the volume fraction g available to the ^{3}He atoms is approximately the same as the number fraction g'. This is a good approximation, as the effective volumes of ^{3}He and ^{4}He are close enough within the other approximations of my calculations. One could easily carry both g and g' through in the analysis.

Thus, in this case the Fermi energy is still given by equation (3), since the corrected number density η of ^{3}He results to first approximation in the overall number density n. However, λ is related to the number density n_3 of ^{3}He, so that equation (1) becomes

$$\lambda \geq 4d = 4n_s^{-1/3} = 4\left(\frac{kT_c}{E_F} n_3\right)^{-1/3} = 4\left(\frac{kT_c}{E_F} gn\right)^{-1/3} \tag{6}$$

Combining equation (6) with (2)–(4), we get the superfluid transition temperature for a dilute Fermi liquid,

$$T_{dF} \leq \frac{h^2 n^{2/3} g^2}{(8f)^3 m_e k} \tag{7}$$

For a 5% concentration of ^{3}He atoms in L^{4}He, equation (19) predicts $T_{dF} \sim 10^{-5}$ K for $m_e \approx m_{^3\text{He}} \approx 5 \times 10^{-24}$ g. If $m_e \sim 10 m_{^3\text{He}}$ for a quasiparticle of one ^{3}He surrounded by ^{4}He, $T_{dF} \sim 10^{-6}$ K. This is significantly higher than more elaborate theories that put this transition at $\sim 10^{-9}$ K. Experiments with dilute solutions are under way, but no superfluid transition has yet been observed for the solute ^{3}He atoms.

5. ANALYSIS FOR HELIUM-4

It will next be shown that the same paradigm applies to the superfluid transition temperature of liquid ^{4}He (L^{4}He), but results in a slightly different equation that also agrees well with experiment. In L^{4}He, pairing of the atoms is not necessary, as these particles are already bosons, and there is no Fermi energy below which all the states are filled for fermions because of the Pauli principle. Thus, all the particles may participate in a BE condensation, not just a fraction kT_c/E_F as in the case of fermions.
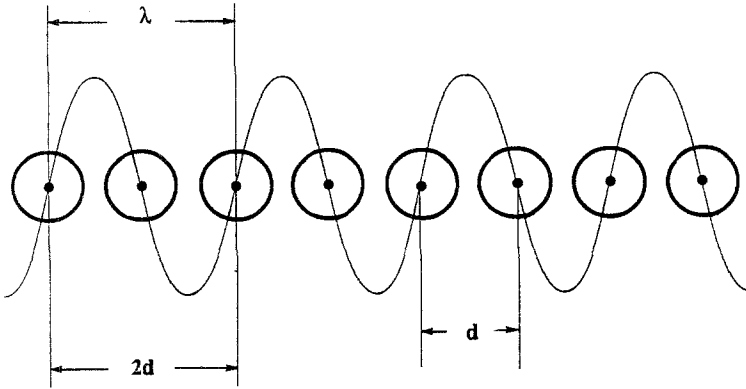
Fig. 2. ^{4}He bosons.

As shown in Figure 2, since pairing does not occur, $\frac{1}{2}\lambda$ encompasses the centers of two bosons when

$$\lambda \geq 2d = 2n^{-1/3} \tag{8}$$

where n is the number density of bosons (^{4}He in this case),

$$\lambda = \frac{h}{p} = \frac{h}{[2(m_e)(\frac{1}{2}fkT_c)]^{1/2}} \tag{9}$$

where the $2m_e$ in equation (4) has been replaced by m_e since pairing is not necessary.

Combining equations (8) and (9), we find that the superfluid transition temperature for a boson gas is

$$T_{cB} \leq \frac{h^2 n^{2/3}}{4fm_e k} \tag{10}$$

For three-dimensional L^{4}He, $f = 3$. Atkins (1959) gives the density of L^{4}He as 0.1465 g/cm^3 at $T \leq 2.17$ K (λ point). Thus, $n \approx 2.2 \times 10^{22}$/cm^3. Taking $m_e \approx m_{^4\text{He}} \approx 6.7 \times 10^{-24}$ g, equation (10) gives

$$T_{cB} \approx 3 \text{ K}$$

in good agreement with the experimental value of 2.17 K for the superfluid transition of L^{4}He (Atkins, 1959).

It is interesting to note that for $f = 3$, equation (10) gives

$$T_{cB} \leq \frac{h^2 n^{2/3}}{12 m_e k}$$

This is almost identically the same as the T_c^{BE} obtained from a more rigorous, but also much more difficult derivation as found in textbooks (Huang, 1963) for a BE condensation for pure bosons:

$$T_c^{BE} = \frac{h^2 n^{2/3}}{2\pi mk[\zeta(3/2)]^{2/3}} = \frac{h^2 n^{2/3}}{2\pi mk[2.612]^{2/3}} = \frac{h^2 n^{2/3}}{11.92mk} \tag{11}$$

where $\zeta(x)$ is the Riemann zeta function of x, and $\zeta(\frac{3}{2}) = 2.612 \ldots$. The ratio $T_c^{BE}/T_{cB} = 12/11.92 = 1.007$.

6. DISCUSSION

It is remarkable how well this simple paradigm works over a range of 12 orders of magnitude from 10^{-3} to 10^9 K. It may seem surprising that it works so well, since an interaction between the particles is not part of the paradigm. Perhaps this should not be so surprising as kT_c itself is a measure of the interaction strength and enters into the equation in different functional forms both as an energy cutoff and in the Cooper pair momentum. Also, the BE condensation temperature T_c^{BE} for ^{4}He works quite well, and it does not include an interaction. Equation (11) gives $T_c^{BE} = 3.1$ K, whereas the experimental value is $T_c = 2.17$ K. The small difference can be attributed to an interaction potential. Possibly it could be attributed to an effective mass $m_e > m_{^4He}$. In any event, only a small correction is necessary. It may well be that there are large interactions, but that their net effect is small.

For each class of materials, interactions can reduce T_c from the ideal values given by my paradigm, so that it basically gives an upper limit of T_c for that class. The quantum condition appears to be the overriding effect in the sense of a general principle that determines the upper limit T_c with little need for microscopic calculations. Three readily stated examples will illlustrate my point.

1. If we wish to find the center of mass of the exhaust gases from a rocket ship in space, we can either do a tedious calculation of the trajectories and momentum transfer of all the $\gg 10^{27}$ particles, or we can simply apply the principle of the conservation of the center of mass in following the motion of the rocket ship.

2. To determine the radiation from a uniformly charged sphere oscillating radially,we could microscopically calculate the radiation from each point on the sphere taking interference into account. Or we could simply note that the center of charge is not accelerating, and hence there is no radiation. Done correctly, either approach will give the right answer; but one way is tremendously more difficult than the other.

3. The diffusion time constant for a magnetic field into a medium of permeability μ and conductivity σ is $\tau = \mu\sigma\delta^2/2$, where δ is the penetration

depth. We could make laborious measurements of μ, σ, and δ to determine τ. On the other hand, knowing that $\delta = [2/\mu\sigma\omega]^{1/2}$ (ω is the angular frequency), we can find τ quite easily: $\tau = 1/\omega$.

If the BCS paradigm applies to dilute ^{3}He, then scattering from ^{4}He should have no effect on the center-of-mass motion of a ^{3}He pair. BCS assumes that each member of a pair has equal and opposite momentum. This guarantees that as one particle is scattered, the other one will move in the opposite direction to conserve the center-of-mass motion.

There may be a hysteretic effect for T_c for some superconductors or superfluids by preventing the pairing of fermions. Apply a large enough magnetic field to keep the fermions unpaired as T is lowered to near 0 K. It is commonly expected that the superstate will return upon removal of the magnetic field. However, the Pauli exclusion principle may inhibit pairing, and the normal unpaired state may remain as a Fermi fluid—equation (2). Direct cooling without a magnetic field leads to the paired superstate.

7. CONCLUSION

Good experiments will, it is hoped, answer the many questions raised in this paper. Past theory in the prediction of T_c in both the arenas of superconductivity and superfluidity has not been too successful. To my knowledge, the T_c of any superconductor or superfluid has rarely been correctly predicted in advance of the measurement. This is in part due to the inherent complexity and difficulty of the previous approaches.

I have great respect for the extant theories of superconductivity and superfluidity. They are great intellectual achievements. Their complexity does not make them erroneous. Neither does the simplicity of my paradigm make it wrong. One hopes that the two approaches can balance each other. There is a need for both. A good theory must accurately portray a large class of observations with a model that has only a few parameters in it. Furthermore, it must make correct predictions regarding prospective findings.

My paradigm is a useful description of quantum fluids that fits the above requirements well. It is not designed to give a microscopic description. That is done well by the existing theories. It does show that a BE type condensation as a general feature common to a broad range of states of matter appears to be as important as any particular interaction mechanism in the transition of both the superconducting and superfluid states from 10^{-3} to 10^9 K. When pairing occurs at $T \geq T_c$, then the interactions and the pairing mechanism may be irrelevant, as the transition is then primarily limited by the condensation temperature.

ACKNOWLEDGMENT

REFERENCES

Atkins, K. R. (1959). *Liquid Helium*, Cambridge University Press, Cambridge.
Bardeen, J., Cooper, L. N., and Schrieffer, J. R. (1957). *Physical Review*, **108**, 1175.
Betts, D. S. (1969). *Contemporary Physics*, **10**, 241.
Brueckner, K. A., Soda, T., Anderson, P. W., and Morel, P. (1960). *Physical Review*, **118**, 1442.
Emery, V. J., and Sessler, A. M. (1960). *Physical Review*, **119**, 43.
Finkelstein, D. (1988). *International Journal of Theoretical Physics*, **27**, 473.
Finkelstein, D. (1989). *International Journal of Theoretical Physics*, **28**, 1081.
Huang, K. (1963). *Statistical Mechanics*, Wiley, New York.
Leggett, A. J. (1975). *Reviews of Modern Physics*, **47**, 331.
Osheroff, D. D., Richardson, R. C., and Lee, D. M. (1972a). *Physical Review Letters*, **28**, 885.
Osheroff, D. D., Gully, W. J., Richardson, R. C., and Lee, D. M. (1972b). *Physical Review Letters*, **29**, 920.
Pitaevskii, L. P. (1959). *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, [*Soviet Physics-JETP*, **10**, 1267 (1960)].
Rabinowitz, M. (1987). In *Proceedings: EPRI Workshop on High-Temperature Superconductivity*, Washington, D.C.
Rabinowitz, M. (1988). In *Proceedings: EPRI Conference on Electrical Applications of Superconductivity*, Orlando, Florida.
Rabinowitz, M. (1989a). *International Journal of Theoretical Physics*, **28**, 137.
Rabinowitz, M. (1989b). *Physica C*, **162-164**, 249.
Rabinowitz, M. (1990). In *Advances in Cryogenic Engineering*, Vol. 36A, Plenum Press, New York, p. 21.
Rodríguez-Núñez, J. J., and Tello-Llanos, R. (1991). *International Journal of Theoretical Physics*, **30**, 857.
Wheatley, J. C. (1975). *Reviews of Modern Physics*, **47**, 415.